

# First-order wave equations on networks

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In the talk we present recent results concerning well-posedness of first-order one-dimensional wave equations on networks. Examples of such equations are, among others, the telegrapher's equation and Saint-Venant shallow water equations. We consider them on networks represented by metric graphs, i.e., graphs with edges identified with the unit interval. The main challenge in such problems is to formulate suitable boundary conditions ensuring well-posedness. We consider local linear boundary conditions coupling together the values of the functions at vertices, i.e., at the endpoints of the unit interval. We present the construction of such conditions and show that the obtained initial-boundary value problem admits unique solutions governed by a  $C_0$ -semigroup.

## References

- [1] J. Banasiak, A. Błoch, *Telegraph systems on networks and port-Hamiltonians. I. Boundary conditions and well-posedness*, *Evol. Equ. Control Theory* 11, 1331–1355 (2022)

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